Semiannual Progress Report No. 6

A STUDY OF SELECTED RADIATION AND PROPAGATION PROBLEMS RELATED TO ANTENNAS AND PROBES IN MAGNETO-IONIC MEDIA

Grant No. NsG 395

The National Aeronautics and Space Administration

Period covered

1 October 1965 to 31 March 1966

Antenna Laboratory
Department of Electrical Engineering
Engineering Experiment Station
University of Illinois
Urbana, Illinois

TABLE OF CONTENTS

[redacted]

INTRODUCTION

This semiannual report covers the activities in the period from October 1, 1965 to March 31, 1966. Most research subjects reported earlier have been continued, some are approaching completion, a few new ones have been added.

SUMMARY OF THE RESEARCH

1. Impedance of an Electric Current Loop in an Anisotropic Cold Plasma - G. L. Duff

Since the last report, the numerical calculation of the input impedance of the loop in a plasma, for the case of the steady magnetic field normal to the plane of the loop, has been completed. Calculations were done for the quasi-static approximation plus a first order correction term, as well as for the case of a uniaxial medium.

After spending much time on the theoretical derivation for the impedance of the loop at an arbitrary orientation to the imposed steady magnetic field, it was decided that it was too difficult a problem to be solved analytically. An expression has been derived, but it is in the form of a five-dimensional integral that seem unworkable.

However, the case of the magnetic field in the same plane as the loop is of interest and should be somewhat simpler than the arbitrary orientation case, so an attempt is being made to analyze that configuration.

2. Experimental Measurement of Loop Impedance in a Magnetoplasma - G. L. Duff

During the past period, experimental measurements of the loop impedance have been made. The present results indicate good qualitative agreement with the theory, and further measurements are planned. Values of X from zero to ten and values of Y up to about two have been achieved. It is somewhat difficult, however, to get a full range of plasma parameters due to the interaction of the steady magnetic field and the electron temperature and density.

If the attempts to calculate the loop impedance for the magnetic field in the plane of the loop are successful, measurements for that case will also be made.

In the meantime, numerical comparison of the experimental and theoretical results will be continued.

3. Wave Propagation in Plasmas with Very Strong Magnetic Fields - S. W. Lee, C. Liang, Y. T. Lo

In the study of wave propagation in a warm magnetoplasma, the fluid equations are usually supplemented by the equation of state where the pressure is assumed to be isotropic. This model appears to be a good approximation only for a collision-dominated regime. In the case of low density and strong static magnetic field the pressure is no longer isotropic. Therefore, in this case the third moment equation derived from the Boltzmann-Vlasov equation is used instead of the equation of state. By following Chew-Goldberger-Low's double adiabatic theory, the dispersion relation for plane wave propagation can be determined. In this medium the electro-acoustic wave travels at different speeds in the longitudinal and transverse directions. Propagation characteristics for a few special cases are studied in more detail. This work has been accepted for publication in Radio Science in the very near future.

4. Current Distribution and Input Admittance of Cylindrical Antenna in a Magnetoplasma - S. W. Lee and Y. T. Lo

The current distribution and the input admittance of an infinitely long cylindrical antenna driven by a slice generator and immersed in an anisotropic plasma are investigated. The applied dc magnetic field is assumed to be parallel to the antenna axis. By superimposing the characteristic waves guided along the antenna, the current solution is obtained in the form of a one-dimensional integral of which the integrand is very complicated. The asymptotic behavior of the current both close to and far away from the feed are examined analytically. Numerical results for the current distribution and input admittance under various conditions also have been obtained. Some important conclusions are summarized below.

(1) When $K_\perp$ is positive where $K_\perp = 1 - X/(1 - Y^2)$, $X = (\omega_p/\omega)^2$, $Y = (\omega_c/\omega)$, ω_p = plasma frequency, and ω_c = cyclotron frequency, the magnitude of the axial current decays slowly with the distance from the source, and its phase is nearly linear with a "propagation constant" equal to $\sqrt{K_\perp}\, k_o$ for an antenna with very small radius. Both the attenuation factor and the "propagation constant" increase with the radius of the antenna.

(2) When $K_\perp$ is negative, the magnitude of the axial current decays rapidly away from the feed, and its phase is no longer linear.

(3) The input admittance decreases with the radius of the antenna. The input susceptance increases as the width of the gap at the feed decreases, while the input conductance remains almost unchanged. For small X and Y, the medium becomes nearly isotropic. The numerical results approach those obtained by Hallen.

During the next period, the current distribution and the input admittance for a finite antenna immersed in an anisotropic plasma will be studied by making use of the results for the infinite antenna.

5. Anisotropic Waveguides - I. Akkaya

The work on anisotropic waveguides is near completion. During this period, many numerical results have been obtained; in particular, the temperature effect has been determined in both unbounded and bounded (i.e., in the waveguide) plasmas by comparing the solutions for the cold and the warm plasma models. It is known that in an anisotropic warm plasma the electromagnetic and acoustic waves are not completely uncoupled. From this investigation it is found that the propagation constants for the warm plasma have essentially the same characteristics as those in a cold plasma model; however, in addition there are many more possible modes with, in some ranges, almost any values of the propagation constant. This is due to the fact that the acoustic wave has very short wavelength. When the propagation constants of these two waves are of the same order of magnitude, strongly coupled hybrid modes take place.

6. Theoretical and Experimental Investigation of Circular Waveguides Partially Filled with Warm Plasma - C. Liang, Y. T. Lo

The source-free field solutions inside a partially-filled plasma waveguide have been obtained, and the study of the propagating characteristics of the fundamental modes are continued. A computer program has been written for the solution of the determinantal equation when $\omega > \omega_p$. We are planning to investigate the surface wave propagating characteristics ($\omega < \omega_p$) during the next interval. Since we are interested in studying the wave behaviors inside a glass-lined waveguide filled with compressible plasma due to a simple source and their effect on the source's impedance, basic theorems such as the Lorentz's reciprocity relations and the orthogonality conditions for an inhomogeneously-filled compressible plasma waveguide have been studied. It is of interest to note that these relationships are valid for a compressible plasma medium only if the rigid boundary condition, $\hat{n} \cdot \bar{u} = 0$, for the electron velocity is imposed. Investigation of field solutions due to a source in this waveguide has been initiated and will be continued.

An order has been placed for the purchase of a 2" complete vacuum system (model VS-9) from the VEECO Corporation. This system is mobile, compactly built, and equipped with vacuum-gauge controls; it is perfectly suited for our experimental investigation. At present, we are searching for a good method to produce a reasonably warm steady-state gaseous discharge whose electron temperature can be altered within a suitable range. Basic plasma diagnostic techniques for obtaining the medium's parameters are also being studied at present.

7. Reciprocity, Equivalence and Huygen's Principle in a Compressible Magnetoplasma - G. A. Deschamps

The extension of equivalence principles from electromagnetic theory to the case of a compressible magnetoplasma has been worked out. This section of the report presents a summary of the results.

7.1 Wave equation and some notations:

The field in a compressible magnetoplasma can be represented by a

10 component vector

$$\varphi = \begin{bmatrix} E \\ H \\ p \\ V \end{bmatrix} \tag{1}$$

where E and H are the electric and magnetic field, p the deviation of electron pressure from its average value and V the velocity vector of the electrons. The vectors E, H, V in this notation must be thought of as column vectors, part of the composite vector φ. The source in general must also be understood as a 10 component vector

$$\psi = \begin{bmatrix} -J \\ -K \\ q \\ f \end{bmatrix} \tag{2}$$

J and K are the electric and magnetic currents (the minus sign is used to simplify the form of the wave equation without departing from conventional notations), q is a scalar representing an eventual source of electrons (they would be injected at a rate of $q\, n_o$ per sec per m^3 in the medium if n_o is the average number density), and f is a force density (pressure due to a moving membrane which would act on electrons).

The wave equation can be written

$$\mathfrak{M} \ \varphi = \psi \tag{3}$$

where $\mathfrak{M}$ is a 10 x 10 matrix which can be decomposed as follows:

$$\mathfrak{M} = (\partial_t - \Omega)\, W + \rho_o + \Gamma_\nabla \tag{4}$$

The matrix W is diagonal

$$W = \begin{bmatrix} \epsilon_o & & & \\ & \mu_o & & \\ & & \sigma_o & \\ & & & \delta_o \end{bmatrix} \tag{5}$$

made up of four constants ϵ_o, μ_o, σ_o, δ_o which describe the medium. The ϵ_o and μ_o are the usual permittivity and permeability of vacuum. The equation $\epsilon_o \mu_o c^2 = 1$ defines the speed of light c. The σ_o and δ_o are relative to the acoustic properties of the electron gas: the density $\delta_o = n_o m$ and the inverse of electricity modulus σ_o. Combined together in the equation $\sigma_o \delta_o a^2 = 1$ they define an acoustic speed a. Since σ_o is related to the temperature T by the equation of state, a^2 is proportional to T

$$a^2 = \gamma \frac{T}{m} \tag{6}$$

if we assume adiabatic compression defined by γ. [To be perfectly correct ϵ_o, μ_o, δ_o should be considered as 3 x 3 diagonal matrices since they have to multiply 3 dimensional vectors.] The matrix W defines the energy density in the medium by the formula

$$w = \frac{1}{2} \varphi^T W \varphi = \frac{1}{2} \epsilon_o E^2 + \frac{1}{2} \mu_o H^2 + \frac{1}{2} \sigma_o p^2 + \frac{1}{2} \delta_o V^2 \tag{7}$$

The terms are easily identified as electric, magnetic, potential (compressional), and kinetic energies. The matrix Ω represents the effect of the constant magnetic field B_o. Introducing the gyrofrequency operator

$$\omega_c = \frac{e}{m} B_o \times \tag{8}$$

the matrix

$$\Omega = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_c \end{bmatrix} \tag{9}$$

The matrix ρ_o depends on the average charge density $\rho_o = - n_o e$ of the electrons and is written

$$\rho_o = \begin{bmatrix} 0 & 0 & 0 & \rho_o \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\rho_o & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

It represents the coupling between optical (or electromagnetic) and acoustical waves.

The matrix Γ_∇ represents the operator of differentiation with respect to space coordinates

$$\Gamma_\nabla = \begin{bmatrix} 0 & -\nabla\times & 0 & 0 \\ \nabla\times & 0 & 0 & 0 \\ 0 & 0 & 0 & \nabla\cdot \\ 0 & 0 & \nabla & 0 \end{bmatrix} \tag{11}$$

the upper left square corresponding to Maxwell's equations, the lower right square to equations of acoustic.

7.2 Poynting theorem

The complete matrix $\mathfrak{M}$ is

$$\mathfrak{M} = \begin{bmatrix} \partial_t \epsilon_o & -\nabla\times & 0 & \rho_o \\ \nabla\times & \partial_t \mu_o & 0 & 0 \\ 0 & 0 & \partial_t \sigma_o & \nabla \\ -\rho_o & 0 & \nabla & (\partial_t - \omega_c)\delta_o \end{bmatrix} \tag{12}$$

The advantage of the decomposition is to show individually the role of the various parts. The matrix Γ_∇, for example, serves to introduce the energy flow vector

$$S = \mathbf{E} \times H + p\ V \tag{13}$$

which generalizes the Poynting vector. It is verified that

$$\nabla \cdot S = \varphi^T \Gamma_\nabla \varphi \tag{14}$$

Forming $\varphi^T \mathfrak{M} \varphi$ we find that the parts $\varphi^T \Omega \varphi$ and $\varphi^T \rho_o \varphi$ reduce to zero because $\Omega^T = -\Omega$, $\rho_o^T = -\rho_o$. We are left with

$$\varphi^T \partial_t w \varphi + \varphi^T \Gamma_\nabla \varphi = 0 \tag{15}$$

from which follows Poynting theorem

$$\partial_t w + \nabla \cdot S = 0 \tag{16}$$

The projection of the energy flow vector S on a unit vector n can be expressed by

$$n \cdot S = \frac{1}{2} \varphi^T \Gamma_n \varphi \tag{17}$$

where Γ_n is obtained from Γ_∇ by substituting n for ∇.

7.3 Single frequency field and plane waves

The quantities φ and ψ, field and source, were considered a function of position x and time t. We can take various Fourier transforms of these quantities with respect to time t or with respect to space x or with respect to both. We shall consider this as a change of representation of the quantity considered and use the same symbol for all these transforms but indicate the representation used either by writing the variable after each symbol: φ (x,t), φ (x,ω), φ (k,t), φ (k,ω) or after each equation. Thus, Equation (3) is

$$\mathfrak{M} \varphi = \psi \qquad (x,t)$$

if $\mathfrak{M}$ is given by (4). We also have

$$\mathfrak{M} \varphi = \psi \qquad (x,\omega)$$

with $\mathfrak{M}$ given by

$$\mathfrak{M} (x,\omega) = (j\omega - \Omega)\, W + \rho_o + \Gamma_\nabla \tag{18}$$

or

$$\mathfrak{M} \varphi = \psi \qquad (k,\omega)$$

with $\mathcal{M}$ given by

$$- j \mathcal{M} (k,\omega) = (\omega + j \Omega) W - j \rho_o - \Gamma_k \qquad (19)$$

The latter equation defines for a fixed (k,ω) and $\psi = 0$, the plane wave solutions of the problem. Matrix (19) is hermitian but not symmetrical.

The determinantal equation is

$$D (k,\omega) = \det \mathcal{M} (k,\omega) = 0 \qquad (20)$$

It defines the dispersion surface. The Equation (20) can be simplified by introducing the operator $Y = \frac{\omega_c}{\omega}$, the parameter $X = \frac{\varpi^2}{\omega^2}$ where $\varpi^2 = \frac{\rho_o^2}{\epsilon_o \delta_o}$ and two indices. $n_a = \frac{k}{\omega} a$, $n_c = \frac{k}{\omega} c$ (obviously related).

$$\det \left\{ (1 - n_c^2 k^{\perp})(1 + jY) - n_a^2 k^{=} - X \right\} = 0 \qquad (21)$$

The symbols $k^{=}$ and $k^{\perp}$ represent projection operators on the direction of k and the direction perpendicular to it.

7.4 Cross flux and reaction

In order to generalize the integral form of the Lorentz reciprocity theorem it is necessary to define the reaction between a field φ_1 and a source ψ_2. The proper definition is the integral of a product $\varphi_1^t \psi_2$ taken over the support of the source distribution ψ_2. The φ^t is a modified transpose of φ defined by

$$\varphi^t = [\ E^T \quad - H^T \quad p \quad - V^T\ ] \qquad (22)$$

(The reason for the minus signs can be traced back to the effect of time reversal operation.)

The reaction of the field φ_1 on the sources contained within the volume V will be denoted by $< \varphi_1 \ V \ \psi_2 >$ and is defined by

$$< \varphi_1 \ V \ \psi_2 > = \int_V \varphi_1^t \ \psi_2 \ dv \qquad (23)$$

A cross-flux of two fields φ_1 and φ_2 through the oriented surface S will be defined by

$$\int_S (E_1 \times H_2 - E_2 \times H_1 + p_1 V_2 - p_2 V_1) \cdot n \, ds \tag{24}$$

and denoted by $< \varphi_1 \; S \; \varphi_2 >$.

With these notations it is found that for any volume V and its boundary $S = \partial V$ we have

$$< \varphi_1 \; \partial V \; \varphi_2 > = < \varphi_1 \; V \; \psi_2 > - < \varphi_2 \; V \; \psi_1 > \tag{25}$$

In this formula φ_1 is the field produced by the sources ψ_1 in a given medium and φ_2 the field produced by ψ_2 in the medium where the magnetic field B_o is reversed. Thus, in the (ω, x) representation

$$\mathcal{M} \; (x, \omega; \; \Omega) \; \varphi_1 = \psi_1$$
$$\mathcal{M} \; (x, \omega; \; - \Omega) \; \varphi_2 = \psi_2$$

The relation (23) is the generalization of the integral form of Lorentz relation. The notation chosen to agree with that of reference 4 suggests that a number of consequences derived there are applicable to fields and sources in compressible magnetoplasmas. We shall only indicate the application to equivalence theorems.

Huygen's principle

The problem solved by Huygen's principle is to express the field everywhere inside a source-free region in terms of the field on the boundary of this surface. If n is the unit vector normal to that surface pointing outward from V, the surface distribution of source is simply

$$\Gamma_n \; \varphi$$

It does involve the four types of sources: electric, magnetic, electronic and mechanic. It produces a zero field outside of the volume V.

Other equivalences

By modifying the region outside of V, in particular by introducing a wall on ∂ V it is possible to express the inside field in terms of a smaller number of components of the field. This should be expected since those components are not independent.

A wall is defined by some boundary conditions. We shall use the following notations. An E-wall is a perfect conductor which forces E_{tan} to be zero. An H-wall would force H_{tan} to be zero. A V-wall, called a hard wall in acoustic, imposes the condition $V_{normal} = 0$. A p-wall, or soft wall, imposes the condition p = 0.

We may consider combinations of these conditions. Usually, a conductor is supposed to force the normal velocity as well as the tangential electric field to be zero. It would be an EV-wall in our notation. If some ratio of two quantities is imposed on S we shall call it an impedance wall. For instance, we may have a (E/H = Z)-wall or a (E/V = Y)-wall.

To fully deserve the name of "wall" the surface condition should force the Poynting vector to be zero in the direction n. Thus,

$$n \cdot \bar{S} = \frac{1}{2}\varphi^{+}\,\Gamma_{n}\,\varphi = 0 \qquad (\omega, x)$$

(In the (ω,x) representation the hermitian transpose φ^{+} should be used to give the average Poynting vector $\bar{S}$.) This will be true for an EV-wall, an Ep-wall, an HV and an Hp-wall but not for an E-wall, for instance.

An equivalence can be derived even if the wall is not perfect. It can be put to use provided we know the Green's function in the presence of this boundary.

7.5 Conclusions

The principles here derived should help in the solution of radiation and diffraction problems in a compressible magnetoplasma in the same way as they do in classical electromagnetics.

Publications on closely related subjects are the papers by M. Cohen[2] which are, however, limited to isotropic plasma and those of Marcuvitz[3] and Deschamps[4] advocating the abstract approach. Before those came the basic papers of Rumsey[1] and of course Lorentz.

This study will be continued by the preparation of a more complete report and discussion of some applications.

References

1. Rumsey, V. H., "Reaction Concept in Electromagnetic Theory," Phys. Rev., 94, 1483-1491, 1954.

2. Cohen, Marshall M., "Radiation in a Plasma," (3 parts), Phys. Rev., 123, 711, 1961; 126, 389, 1962; 126, 398, 1962.

3. Marcuvitz, N., "Abstract Operator Methods on Electromagnetic Diffraction," Electromagnetic Waves, (ed. B. E. Langer) University of Wisconsin Press, 1962.

4. Deschamps, G. A., "Scattering Diagrams in Electromagnetic Theory," Electromagnetic Waves, (ed. E. C. Jordan) Pergamon Press, 1963.

8. Radiation of an Antenna in a Compressible Magnetoplasma - Georges A. Deschamps* and O. B. Kesler**

(Abstract of paper to be presented at URSI spring meeting in Washington, D.C.)

The far field produced by arbitrary sources in a compressible magnetoplasma is expressed in terms of the characteristic plane waves in that medium.

The field may be considered as a vector ϕ with ten components, made up of the electromagnetic field and the velocity-pressure field of the electrons. A characteristic wave ϕ_k is entirely defined, except for a constant factor, by its propagation vector k. The extremity of k must lie on the dispersion surface S (also called refractive index surface). The arbitrary factor being disposed of by normalization, the far field at point r is expressed as

$$\phi\ (r) = \Sigma\, a\ (k)\ \phi_k(r)$$

where the sum is taken over the points k of S where the outward normal is in the direction of the vector r.

* Antenna Laboratory, Department of Electrical Engineering, University of Illinois, Urbana, Illinois.

** University of Texas, Austin, Texas.

The coefficient a(k) in this representation is proportional to the radius of Gaussian curvature of S at the point k and to the reaction, properly defined, of the given source with characteristic plane waves in the transpose medium (static magnetic field reversed). This reaction itself can be expressed as a scalar product of the characteristic field $\phi_k(0)$ at the origin, a known vector, and of the Fourier transform of the source distribution evaluated at point k.

The result includes the fields produced by the electromagnetic as well as the acoustic sources. The solutions for a cold magnetoplasma and for a compressible plasma with magnetic field, which have been described before (1,2,3,4), are obtained as special cases.

The research reported in this paper was sponsored by the National Aeronautics and Space Administration under Grant NSG 395.

References

1. Arbel, E. and Felsen, L. B., "Theory of Radiation from Sources in Anisotropic Media," Electromagnetic Theory and Antennas, (ed. E. C. Jordan) Pergamon Press, New York and London, 391-459, 1963.

2. Deschamps, G. A. and Kesler, O. B., "Radiation Field of an Arbitrary Antenna in a Magnetoplasma," IEEE Trans., AP-12, 783-785, November 1964.

3. Kuehl, H. H., "Excitation of Waves in a Warm Plasma by a Current Source," J. Phys. Fluids, 6, No. 10, 1465-1469, October 1963.

4. Seshadri, S. R., "Radiation from Electromagnetic Sources in a Plasma," IEEE Trans., AP-13, 79-88, January 1965.

9. Publications Under Grant No. NsG 395 During this Period

1. Kesler, O. B., "Propagation of EM Waves in Linear, Passive, Generalized Media," Antenna Laboratory Technical Report No. 65-9, University of Illinois, October 1965.

2. Lee, S. W., Lo, Y. T. and Mittra, R., "Finite and Infinite H-Plane Bifurcation of a Waveguide with Anisotropic Plasma Medium," Canadian Journal of Physics, 43, pp. 2123-2135, December 1965.

3. Lee, S. W. and Lo, Y. T., "Radiation in an Anisotropic Moving Medium," Radio Science (New Series), No. 1, March 1966.

4. Lee, S. W., "Guided Waves in Plasma Media," Ph.D. thesis, directed by Y. T. Lo, January 1966.

5. Lee, S. W., Liang, C. and Lo, Y. T., "Wave Propagation in Plasma with Very Strong Magnetic Field," soon to appear in Radio Science.

6. Lo, Y. T. and Lee, S. W., "Current Distribution and Input Admittance of Cylindrical Antenna in Anisotropic Plasma," a summary, soon to appear in IEE Electronics Letters.

7. Deschamps, G. A. and Kesler, O. B., "Radiation of an Antenna in a Compressible Magnetoplasma," the paper is being prepared for publication and will be presented at the URSI Spring Meeting in Washington, D. C. (Abstract included).

8. Deschamps, G. A., "Radiation d'une Antenne dans un Milieu Anisotrope," L'onde Electrique, No. 465, pp. 1379-1385, December 1965.

10. Travel

In June, 1965, Prof. G. Deschamps attended the "Colloque Sur les Antennes en Milieu Ionisé" organized by the Centre National d'Etudes Spatiales in Paris and presented a paper now published in l'Onde Electrique, Vol. 45, pp. 1379-1385, December 1965.

Prof. R. Mittra visited the plasma laboratories at the Universities of Stockholm, Gothenberg and the United Kingdom Atomic Research Laboratories in Culham, England, during his sabbatical leave of absence from September, 1965 to February, 1966. He presented papers at the EM Theory Symposium in Delft and Plasma Conference in London. He did some study in the area of plasma instabilities during his stay at the University of Oxford.

11. Financial Information

Financial information is contained in the Quarterly Financial Reports submitted by the University of Illinois Business Office on Form No. 1030.